S

93201Q

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

New Zealand Scholarship

Statistics and Modelling, 2004

9.30 am Tuesday 16 November 2004
Time allowed: Three hours

QUESTION BOOKLET

You should answer ALL the questions in the separate Answer Booklet No. 93201A.

The approximate time you should spend on each question is as follows:

Question	Time (minutes)
1	20
2	25
3	40
4	40
5	25
6	30

A Mathematical Formulae and Tables Booklet S–STATF is provided in the centre of this examination paper.

Check that this question booklet has pages 2–7 in the correct order, as well as a centre pull-out of 4 pages of mathematical formulae and tables, and that none of these pages is blank.

Scholarship Criteria
The student will:
- demonstrate an advanced level of statistical thinking
- apply probability theory and models to solve complex problems
- apply techniques of mathematical modelling to solve complex problems.

Scholarship with Outstanding Performance Criteria
In addition to meeting the criteria for Scholarship the student will:
- demonstrate sophistication of thinking across a range of concepts in the solution of problems
- in their solutions display logical development, precision and clarity across a range of concepts
- critically evaluate processes and solutions.

YOU MAY KEEP THIS QUESTION BOOKLET AT THE END OF THE EXAMINATION.

© New Zealand Qualifications Authority, 2004
All rights reserved. No part of this publication may be reproduced by any means without the prior permission of the New Zealand Qualifications Authority.

All questions are about a large hardware store named NAILS located in Auckland. This store has a gardening department and belongs to a New Zealand-wide chain of hardware stores.

QUESTION ONE

A survey was conducted by NAILS to determine the level of customer support for a proposed garden waste disposal service. A random sample of 200 customers was interviewed and 46 indicated their support for this proposal.

(a) Find a 95% confidence interval for the percentage support of all customers for this proposal.
(b) Assuming that the percentage support of all customers is the value given by the lower limit of the interval in part (a), calculate the probability that the level of support from another sample of 200 customers is at least 25%.
(c) NAILS claims that at least 30% of all customers support this proposal. Can this claim be justified? Give a reason for your answer.
(d) The manager of NAILS requests that a second survey be conducted so that the width of the interval in (a) is reduced by 40%. How many customers should be interviewed in the second survey?

QUESTION TWO

The gardening department in NAILS makes three types of compost: azalea, bulb and all-purpose to suit customers' requirements. To make these types of compost, nitrogen, phosphorus and potassium are added in different proportions. The ratio of nitrogen : phosphorus : potassium for the three types of compost is as follows: azalea 1:1:1, bulb 4:5:6 and all-purpose 5:6:7. For a particular week's production there are 5.3 tonnes of nitrogen, 8.4 tonnes of phosphorus and 11.5 tonnes of potassium available.

Using this information, a system of equations can be formed to calculate the amount of each type of compost able to be manufactured, using all the nitrogen, phosphorus and potassium available. If a, b and c represent respectively, in tonnes, the amount of azalea, bulb and all-purpose compost manufactured during this week, then two of these equations are:

$$30a + 24b + 25c = 477$$
$$30a + 30b + 30c = 756$$

(i) Find the third equation.
(ii) Find the general solution for this system of equations in terms of c.
(iii) Comment on the practical consequences of the solution in part (ii).

QUESTION THREE

A market research company is contracted by the general manager of NAILS to investigate the effect of promotional expenditure on sales of plants. The results of the investigation will be used to make sales predictions for all gardening departments in the New Zealand-wide store chain.

For each month from January 2001 to December 2002, the amount spent on promoting plant sales (promotional expenditure) and the total plant sales by the chain were recorded. The data, a scatter plot and some statistical output relating to the data are shown below. E represents the promotional expenditure for the month (in thousands of dollars, \$000) and S represents the total plant sales for the month (in thousands of dollars).

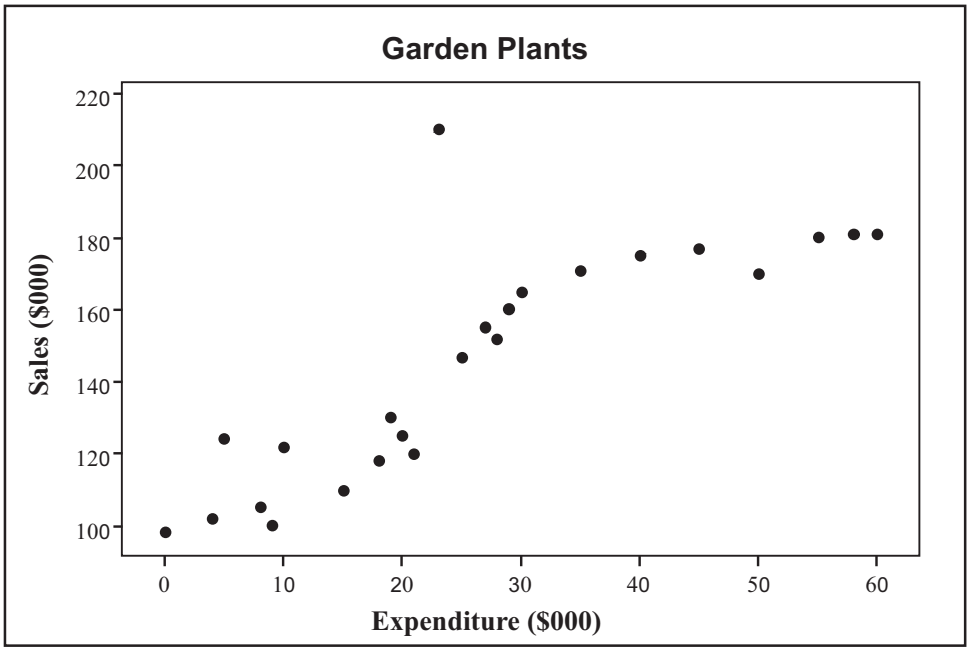
Write a report (approximately one page long) to the general manager of NAILS that summarises the statistical output given below. Include some sales predictions in your report.

Data Output for Question Three

Expenditure (\$000)	Sales (\$000)
0	98
4	102
5	124
8	105
9	100
10	122
15	110
18	118
19	130
20	125
21	120
23	210
25	147
27	155
28	152
29	160
30	165
35	171
40	175
45	177
50	170
55	180
58	181
60	181

Value of E	Correlation r	Regression Equation
All values	0.819	$S = 105 + 1.51E$
All less outlier	0.928	$S = 101 + 1.54E$
$E \geq 15$ less outlier	0.885	$S = 104 + 1.47E$
$15 \leq E \leq 30$ less outlier	0.972	$S = 53.2 + 3.67E$

Scatter Plot



QUESTION FOUR

NAILS is planning to produce and market two different weed killers, POW and ZAP. Limitations on the use of resources impose the following constraints on the production of POW and ZAP.

- To produce one bottle of POW takes 30 minutes and to produce one bottle of ZAP takes 20 minutes, and there are 25 hours available in total for this production-run operation.
- Availability of a particular additive means that the chain cannot produce more than 35 bottles of POW and a combined total of 65 bottles on any production run.
- As a minimum production requirement, at least 15 bottles of each weed killer must be produced in each run.
- POW and ZAP cannot be produced simultaneously.

(a) The management of NAILS wants to know how many bottles of both POW and ZAP should be produced. The preliminary estimates of their potential profitability are \$20 per bottle of POW and \$10 per bottle of ZAP. Perform an appropriate analysis and make a recommendation to management about the amount of POW and ZAP that could be produced to maximise profit.

(b) Suppose that the profit of \$20 per bottle of POW was an overestimate and it was in reality only \$15 per bottle. How does that change your production recommendation in part (a)?

It is found that the sales of POW and ZAP are closely related by the function $y = 9\ln(x)$,
where x = number of bottles of POW and y = number of bottles of ZAP.
Management therefore requests that the ratio "number of bottles of POW produced : number of bottles of ZAP produced be $x : 9\ln(x)$". The constraints need to be satisfied and the estimated profit needs to be maximised, based on a profit of \$15 per bottle of POW and \$10 per bottle of ZAP, so as to mirror demand.

(c) Suppose x is the number of bottles of POW that should be produced to meet management's request and maximise the profit. Find an implicit equation for x.

(d) Solve the equation derived in part (c) and make a recommendation to management about the amount of POW and ZAP that could be produced to maximise profit.

QUESTION FIVE

Plants are delivered to NAILS in punnets (small containers) by truck. Each truckload has approximately 250 punnets. The following sampling scheme is used by NAILS to check on the quality of these punnets.

Scheme 1

A random sample of 20 punnets is chosen from the truckload. If there are fewer than 2 defective punnets in the sample, then the truckload is accepted. If there are at least 2 defective punnets in the sample, then the truckload is rejected.

(a) Two percent of punnets from one supplier, *Grow Well*, are known to be defective.

1. Two truckloads of punnets from *Grow Well* arrive at NAILS. Calculate the probability that exactly one truckload of the two is accepted.

2. Under **Scheme 1**, *Grow Well* tries to achieve a target acceptance rate of at least 96% of all truckloads. To the nearest 0.5%, find the greatest percentage defective rate for punnets that would enable *Grow Well* to achieve its target acceptance rate.

3. Suppose that the following two-stage sampling scheme was used.

Scheme 2

A random sample of 20 punnets is chosen from a truckload. If there are fewer than 2 defective punnets in the sample, then the truckload is accepted. If there are at least two defective punnets, then a second sample of 20 punnets is chosen. If there are no defective punnets in the second sample, then the truckload is accepted, but if there are any defective punnets in the sample, the truckload is rejected.

Under **Scheme 2**, calculate the proportion of accepted truckloads that would be accepted on the second sample.

(b) Another plant supplier, *Quality Plants*, has a rate of 1 defective punnet in 150. Suppose that the following sampling scheme was used.

Scheme 3

A random sample of n punnets is chosen from a truckload. If there are fewer than 2 defective punnets in the sample, then the truckload is accepted. If there are at least 2 defective punnets in the sample, then the truckload is rejected.

Find the largest value of n that would achieve an acceptance rate of at least 91% (to the nearest 1%) under **Scheme 3**, of all truckloads from *Quality Plants*.

QUESTION SIX

The value of the retail sales at NAILS was recorded each day between Monday 1st January 2001 and Friday 30th April 2004.

You have been provided with **some** of the resulting time series data, which are shown in the tables and graphs on this page and page 7.

Table 1 shows the monthly retail sales from July 2001 to October 2003 inclusive. Centred moving means have been calculated and are also shown in the table. For these data, the least squares regression line fitting the centred moving means was obtained.
The equation of the regression line is $y = 0.91x + 225.21$, where x represents the number of months since June 2001 (so $x = 1$ corresponds with July 2001, etc) and y represents the values of sales (in thousands of dollars, $000).

Table 2 shows the daily retail sales for the five weeks from Sunday 28th September 2003 until Saturday 1st November 2003 inclusive; moving means of order seven have been calculated.
Graph 1 shows the value of the monthly retail sales from 1st July 2001 to 31st October 2003. Centred moving means of order twelve have also been plotted on the graph.

Graph 2 shows the value of the daily retail sales for the five weeks from Sunday 28th September 2003 until Saturday 1st November 2003 inclusive. Moving means of order seven have also been plotted on the graph.

(a) To improve sales, the duty manager for each day is offered a bonus payment if the sales for that day exceed the expected value by at least 5%.
  (1) What features of the time series should be considered in setting up this bonus payment scheme for managers?
  (2) Explain how the daily sales targets could be calculated.

(b) Labour Day, the last Monday of October, results in a three-day weekend which is traditionally used for home renovation and gardening. This produces high sales for hardware stores throughout New Zealand at this time.
  (1) What effect does the high sales on Labour Day 2003 have on the (centred) moving mean for the daily retail sales?
  (2) How would you allow for the high sales figures for Labour Days in the calculation of a sales forecast for Mondays?

(c) Using the given information, forecast the sales for Tuesday 7th December 2004. You must make clear the method you are using to make your forecast and justify your reasoning.

(d) Describe two limitations of the forecast you made in part (c).

TABLE 1

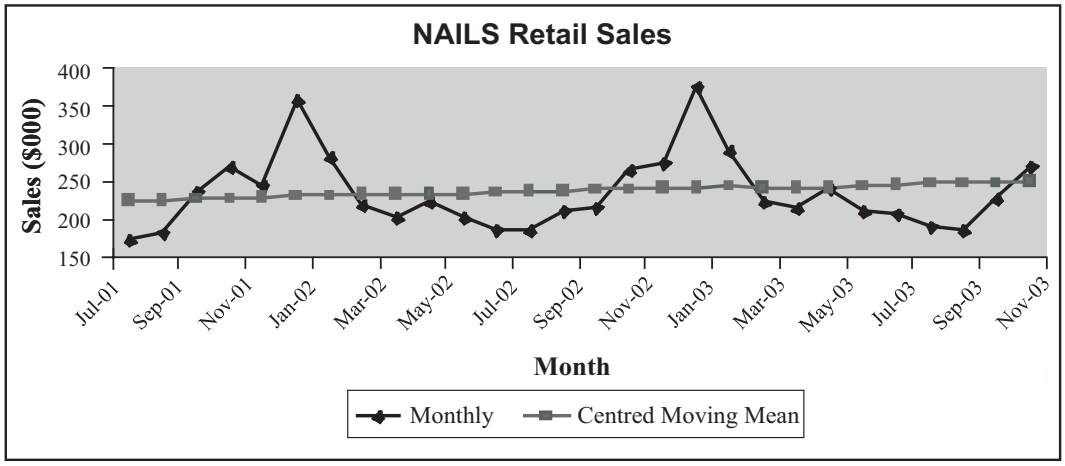
Month	Sales ($000)	Centred Moving Mean
Jul-01	171.8	225.4
Aug-01	182.3	225.9
Sep-01	237.2	226.7
Oct-01	270.3	228.2
Nov-01	245.5	229.5
Dec-01	357.3	230.8
Jan-02	281.2	232.3
Feb-02	218.1	234.2
Mar-02	202.3	234.6
Apr-02	223.6	233.5
May-02	202.9	234.6
Jun-02	188.1	236.7
Jul-02	185.9	237.8
Aug-02	213.3	238.5

Month	Sales ($000)	Centred Moving Mean
Sep-02	216.2	239.2
Oct-02	265.3	240.5
Nov-02	276.4	241.6
Dec-02	376.3	242.9
Jan-03	290.1	244.0
Feb-03	224.1	243.0
Mar-03	214.0	242.3
Apr-03	243.0	243.0
May-03	211.1	244.9
Jun-03	209.3	247.0
Jul-03	190.9	247.4
Aug-03	185.9	249.0
Sep-03	226.7	249.9
Oct-03	272.2	250.6

TABLE 2

Day	Date	Sales ($000)	Moving Mean
Sun	28-Sep	8.9	6.5
Mon	29-Sep	7.4	6.7
Tues	30-Sep	5.4	6.7
Wed	1-Oct	3.7	6.7
Thur	2-Oct	6.3	6.5
Fri	3-Oct	6.2	6.4
Sat	4-Oct	9.0	6.5
Sun	5-Oct	7.7	6.5
Mon	6-Oct	6.2	6.3
Tues	7-Oct	6.1	6.2
Wed	8-Oct	3.9	7.0
Thur	9-Oct	5.0	8.8
Fri	10-Oct	5.7	8.6
Sat	11-Oct	14.1	8.6
Sun	12-Oct	20.6	8.7
Mon	13-Oct	5.0	8.8
Tues	14-Oct	5.7	9.2
Wed	15-Oct	4.8	9.9
Thur	16-Oct	6.0	8.7
Fri	17-Oct	8.0	9.0
Sat	18-Oct	19.0	9.1
Sun	19-Oct	12.6	9.3
Mon	20-Oct	6.6	9.4
Tues	21-Oct	6.7	9.4
Wed	22-Oct	6.2	9.2
Thur	23-Oct	6.6	10.1
Fri	24-Oct	8.4	12.7
Sat	25-Oct	17.1	12.6
Sun	26-Oct	18.9	12.3
Mon	27-Oct	24.7	11.9
Tues	28-Oct	6.2	11.7
Wed	29-Oct	3.9	12.2
Thur	30-Oct	4.1	11.5
Fri	31-Oct	7.2	8.9
Sat	1-Nov	20.2	8.9

GRAPH 1



GRAPH 2